1. Lessons shown in video

Lesson 1 is shown in "Lesson 1: Clip 1". Lesson 3 is shown in both "Lesson 3: Clip 1" and "Lesson 3: Clip 2".

2. Promoting a positive environment

As I circulate the room and interact with students in my two lessons, I show respect for my students by asking them questions politely (demonstrated in Lesson 1: Clip 1 at 0:13, 0:28, 1:20, 1:45, 3:30, 5:45, 6:00, 6:10, 6:50 and Lesson 3: Clip 1 at 0:10, 5:28, 6:20, 6:33). After questioning students, I wait patiently for a response (demonstrated in Lesson 1: Clip 1 at 0:30, 1:30, 1:55, 5:50 and Lesson 3: Clip 1 at 0:13, 1:00, 6:10, 6:49). I use positive statements as I observe work to reinforce their progress and encourage them to continue engaging in the learning. For answers on the right track, I provide praise, and for answers on the wrong track, I use leading questions errors (demonstrated in Lesson 1: Clip 1 at 0:47, 2:15, 5:00 and Lesson 3: Clip 1 at 1:00, 5:20, 5:40, 6:30). I provide encouragement for students with low confidence and who often have correct ideas, but are uncertain (demonstrated in Lesson 1: Clip 1 at 1:20, 5:28). I also create a safe environment for my capable, but low-confidence students by scaffolding with leading questions and support from other students to encourage them to verbalize thoughts (demonstrated in Lesson 3: Clip 1 at 6:00, 6:30). I demonstrate respect by praising students for working well together (demonstrated in Lesson 1: Clip 1 at 0:01, 7:50). It is important that while I question my students to gain a further understanding I respect them by actively listening and asking clarifying questions (demonstrated in Lesson 1: Clip 1 at 0:47, 1:16, 2:48 and Lesson 3: Clip 1 at 1:30, 6:30, 6:40).

The rapport I have with students is clear because they are comfortable discussing their reasoning with me and they take the time to elaborate on their thinking without fear of judgment (demonstrated in Lesson 1: Clip 1 at 1:00,1:31, 3:30, 5:25, 5:50, 7:00 and Lesson 3: Clip 1 at 0:20,1:00, 5:00, 5:50). For example, when I check in with a group they openly admit their disagreement and are amenable sharing their reasoning for how to model the problem (demonstrated in Lesson 3: Clip 1 at 0:48, 1:00, 1:25, 2:25). To challenge students to engage in the learning I use collaborative learning and have students assigned roles, as seen by a student playing the role of the facilitator at 4:10 in Lesson 3: Clip 1 (see PowerPoint 3 for role descriptions). Additionally, I know that certain students need more attention and immediate responsiveness, so I address real world concepts to deepen their understanding (demonstrated in Lesson 3: Clip 1 at 1:30 and during the discussion is Lesson 3, which is not shown). As I work with each group, I gauge how the composition of student ability in the table groups might affect what type of probing or leading questions students need (demonstrated in Lesson 1: Clip 1 at 1:00, 1:42, 2:10, 5:50 and Lesson 3: Clip 1 at 5:12, 5:28 6:28). For example, I know that one student needs prompting to keep on task and I know that another student benefits from leading questions to encourage him to engage in the learning (demonstrated in Lesson 1: Clip 1 from 4:10 to 4:50). Furthermore, at each table, I encourage students to work together in order to evaluate each other's thinking and to challenge students to be active learners throughout the class (demonstrated in Lesson 1: Clip 1 at 0:30, 1:00, 3:30 and Lesson 3: Clip 1 at 0:25, 1:00). The format for this lesson promotes student voice and I demonstrate a confidence in my students' abilities and a mutual respect by encouraging students to do most of the mathematical reasoning, while I simply ask the questions (demonstrated in Lesson 1: Clip 1 at 1:00, 3:30, 6:30 and Lesson 3: Clip 1 at 0:48, 2:00, 5:15, 6:40).

Part of how I maintain a positive learning environment is that students understand the protocol for constructive interactions and shared thinking. In our class, we refer to being fair to each other as equity (Lesson 1: Clip 1 at 7:25). In both lessons, students have a chance to practice collaboration and display mutual respect (demonstrated in Lesson 1: Clip 1 at 0:25, 5:50 and Lesson 3: Clip 1 at 0:25, 0:48, 1:00, 4:22). Additionally, I am responsive to students with varied needs and backgrounds by supporting collaborative learning in which students develop interpersonal skills; thus, students learn to support each other as a supplement to the assistance that I am able to give. For example, in Lesson 3:

Clip 1 at 4:25 and 5:48 the facilitator guides her peer with tutoring and Socratic questioning on how to use the constant multiplier to predict future values. This purposeful juxtaposition benefits both young women because the facilitator reinforces her own learning and the tutee gets individual attention from a peer. The structure of the activity engages all students in the learning because they are responsible for contributing to the group and sharing their multiple perspectives (demonstrated in Lesson 1: Clip 1 at 1:16, 3:30, 7:0 and Lesson 3: Clip 1 at 1:00, 2:00, 3:36, 5:40).

3. Engaging students in learning

My strategy to elicit students' expression of their understanding of the learning target includes facilitating discussions of the meaning of the learning target (discussion from Lesson 3 is not shown due to student privacy). The group problem in both Lesson 1 and Lesson 3 guides students though an activity that demonstrates developing mastery of the learning target when completed correctly. Additionally, I probe students to determine their understanding of how the "starting value" and "rate growth/decay" affect the graph. I follow this up with a detailed exit ticket (Formative Assessment 1.3) that prompts: (1) Rate your confidence level from 1 to 5 (with 5 being the highest) on your level of mastery of the learning target, (2) What parts of the learning target did you struggle with, or do you want to improve upon?, (3) Why is it important to master this learning target? (4) What resources could you use to improve your skills and knowledge related to mastering the learning target? (demonstrated in Lesson 3: Clip 2 from 0:00 to 1:21). This allows students to provide both verbal and written expressions of understanding the learning target and why it is important.

Engaging students with concepts, fluency, reasoning

I use many strategies to engage students in developing conceptual understanding, procedural fluency, and mathematical reasoning and problem solving skills. The lessons require students to complete authentic (or at least culturally relevant) mathematical modeling problems that require all three of the aforementioned understandings and skills to accomplish. For example, in my first lesson I gave students a problem about bug population growth, which requires students to use prior procedural knowledge of how to create and read a table. Also, the students are able to use a prior conceptual knowledge of ratios and geometric sequences to complete the table (demonstrated in Lesson 1: Clip 1 at 0:18, 1:00, 3:30, 6:00). In particular, I had students develop new conceptual understanding by giving them an opportunity ask each other questions and revise their own understandings based on the discussion (demonstrated in Lesson 1: Clip 1 at 3:35 and Lesson 3: Clip 1 at 1:00, 3:30, 4:30,4:50). I structured Lesson 1 and Lesson 3, so that I can assess conceptual understanding of new information through verbal interactions as well as visual observations of the work completed on the desks (demonstrated in Lesson 1: Clip 1 at 0:01, 0:40,1:38, 2:18, 5:50 and Lesson 3: Clip 1 at 0:01, 5:02).

For example, I see in Lesson 3: Clip 1 at 4:40 that the group had a nearly correct table except that the students did not understand that the problem begins with week 0. Because it is largely written on the desks, I was able to address this conceptual misunderstanding with the whole group. Because students work in groups, they have the opportunity to evaluate and revise their thinking as they listen to peers give explanations (demonstrated in Lesson 1: Clip 1 at 3:30). Of course it is critical that students ask each other conceptual questions, so I model throughout the period what types of questions to ask (demonstrated in Lesson 1: Clip 1 at 2:15, 3:20, 4:20, 5:50 and Lesson 3: Clip 1 at 2:35, 3:30, 5:00, 5:50). For example, in Lesson 3: Clip 1 at 5:00, I address the constant multiplier with a group. In Lesson 1, I introduced our Unit Task (not shown), which required students to practice mathematical problem solving by accounting for outliers in real world data. I engage students with multiple opportunities to develop procedural fluency in Lesson 1 and Lesson 3 with tasks containing multiple prompts that require targeted skills and prior knowledge to answer. In Lesson 3, students created tables, graphs, equations, and engaged in discourse by answering questions based on the data (demonstrated in Lesson 3: Clip 1 at 3:33, 6:00). Investigating problems in Lesson 1 and Lesson 3 also requires understanding how to work with exponents, the order of operations, and using arithmetic skills, such as multiplication and subtraction (demonstrated in Lesson 3: Clip 1 at 1:00, 2:25). These activities allowed students at all levels to practice procedural fluency because of the structure of the group and the support provided by other students (demonstrated in Lesson 1: Clip 1 at 3:30 and Lesson 3: Clip 1 at 1:00, 5:50). For example, a student and his group had to recall the correct procedure for calculating a ratio (demonstrated in Lesson 1: Clip 1 at 3:30, 6:30). In addition, students had to recall how to create a graph from ordered pairs on the zombie worksheet in Lesson 3: Clip 1 at 3:33. Furthermore, I affirmed to one student that the procedure we are using is a simplification and I praised her for thinking about more complex procedures needed to model the zombie population (demonstrated in Lesson 3: Clip 1 at 1:30). Frequently in the lessons, I ask students to explain how they got where they are mathematically (demonstrated in Lesson 1: Clip 1 at 0:40, 1:20 7:00 and Lesson 3: Clip 1 at 6:30).

The entire purpose of both Lesson 1 and Lesson 2 was to engage students in problem solving and mathematical reasoning. In Lesson 1, to correctly predict future values of with an exponential model, a student must choose a mathematically valid representation of the data (demonstrated in Lesson 1: Clip 1 at 0:20, 1:10 and Lesson 3: Clip 1 at 0:01, 0:36, 1:20, 3:55, 4:20, 5:12). For example in Lesson 1: Clip 1, as student begins to problem solve by creating variables to represent the quantities in the problem, bugs and days. Additionally students must use reasoning to conjecture that the ratio between the current week and the prior week is. Because the situations presented in Lesson 1 and Lesson 3 had real world implications, we were able to think about how to extend the simple model into real world problem solving and reasoning as demonstrated in Lesson 3: Clip 1 at 1:30 and the closing class discussion in Lesson 3. In the Zombie investigation, student groups had many options choose what approach to use to compare the graphs (visual comparisons of the tables or graphs, calculations of values, comparison of equations). For Lesson 3, students engaged in problem solving with a need to define both possible steps and desired outcomes. In this way, I also differentiated my instruction to allow students to enter the problem at their own level. Furthermore, students worked together to develop shared understanding of the problem demonstrated in Lesson 3: Clip 1 at 2:25, 5:00). Since teamwork is often part of authentic problem solving, I engaged students in a situation that will benefit them beyond simply understanding a mathematical concept. Particularly, in Lesson 1, students were challenged to share their solutions and compare understandings (demonstrated in Lesson 1: Clip 1 at 1:00, 3:47). The second lesson extends the practice of problem solving with a graphic representation. Students must determine how the growth changes with different starting values (the learning target is in Learning Target Expression Clip 3 at 1:00). Students were able to choose what approach to take, which allows different students to compare approaches and to compare and contrast different solution methods (demonstrated in Lesson 3: Clip 1 at 0:50).

Link to prior knowledge and assets

My instruction linked students' prior learning to the lesson in several instances. For example, students had to recall prior procedural knowledge of how to create and read a table. This information was linked to new learning when students had to use the table to find the "constant multiplier" (demonstrated in Lesson 1: Clip 1 at 6:38). Additionally, the students used a prior conceptual knowledge of ratios, geometric sequences, graphs, and equations to complete the variety of questions in Lesson 3 (demonstrated in Lesson 1: Clip 1 at 0:18, 1:00, 1:16, 3:30 and Lesson 3: Clip 1 at 0:48, 2:50, 3:33). For example, students had to recall how to use a ratio to apply it to the process of identifying the "constant multiplier." Additionally students used prior knowledge of our classroom resources and our personal approaches to develop further procedural fluency, as well as problem solving skills (demonstrated in Lesson 1: Clip 1 at 7:25). For example, I had a student recall our classroom norm and mathematical concept of "equity." The concept of equity is one that we also have as a classroom norm, so it is part of our classroom culture. My students were able to take advantage of community assets, such as posters, that both students and I created thus far (demonstrated in Lesson 1: Clip 1 at 7:20).

The instruction in my learning segment is strongly guided by my students' personal, cultural, community assets. In Lesson 3, I used my knowledge student interest in zombies as a context for my learning task (demonstrated in Lesson 3: Clip 1 at 1:00, 5:50, 6:55). For example, groups appear to be motivated and happy to work with the context of zombies (demonstrated in Lesson 3: Clip 1 at 5:45, 6:55). Additionally, I take advantage of the students' shared knowledge of how disease spreads. In the discussion after the zombie task (not shown due to student privacy), we discuss how this model would apply to disease. Using a common knowledge of disease is an asset to providing a common entry point for students in each of the following categories: ELL, underperforming, gifted, and struggling readers. Since I know that several students are interested in pursuing science, I planned for this time to be open-ended. The clips also show that my lessons take advantage of our classroom culture of collaboration (demonstrated in Lesson 1: Clip 1 at 1:00, 3:30, 6:00 and in Lesson 3: Clip 1 at 0:01, 1:35, 4:50). This supports all learners (My three students with Student Learning Plans, high-achieving students, and underperforming students) since students must work together. Additionally, I encourage student to write on our old desks, since most students in my class personally prefer to this medium for collaborating (demonstrated in Lesson 1: Clip 1 at 0:01, 1:00, 6:30 and in Lesson 3: Clip 1 at 0:01). During both lessons, students described their own personal understandings for other students' benefits (demonstrated in Lesson 1: Clip 1 at 3:30, 6:30

and Lesson 3: Clip 1 at 1:00, 4:50). Because student vocabulary is often different from teacher vocabulary, students may be able to understand each other's examples more easily. Finally, I encouraged students to use personal advantages while problem solving by giving them choices to how they approach the problem (demonstrated in Lesson 1: Clip 1 at 0:01, 1:16 and Lesson 3: Clip 1 at 1:00). For example, one student related ratios to "least number of odds" (demonstrated in Lesson1_Clip 1 at 5:59). Choice of approach is a valid structural asset because it removes constraints and allows different students to play to their strengths.

4. Deepen student learning

I elicited and built on student responses to promote thinking throughout the two lessons. In general, Lesson 1: Clip 1 and Lesson 3: Clip 1 contain a frequent use of questioning to direct students to explore the concepts (demonstrated in Lesson 1: Clip 1 at 0:01, 4:45, 5:51, 6:05, 6:50). Additionally, I had students build upon each other's understandings by giving them an opportunity ask each other questions and revise their own understandings based on the discussion (demonstrated in Lesson 1: Clip 1 at 3:35 and Lesson 3: Clip 1 at 1:00, 3:30, 4:30,4:50). When appropriate, I went beyond procedural knowledge and asked open-ended questions, challenging students to elaborate on their responses (demonstrated in Lesson 1: Clip 1 at 2:53). I also elicited student responses to develop procedural fluency. For example, in Lesson 1: Clip 1, I use probing questions to ensure students are applying the correct steps (demonstrated in Lesson 1: Clip 1 at 0:01, 0:25, 2:20, 3:00, 5:50). In Lesson 3: Clip 1 at 5:20, I have a dialogue with students and either affirm the girls' correct thinking or respond to their thinking with a leading question to deepen a group's understanding. Because students write on the table, as in Lesson 3: Clip 1 at 4:40, I am able to verbally or in some cases with my own marker, contribute to the ideas on the table. Since I play the role of a facilitator, I focus on modeling the types of questions students should ask each other, and evidence of students emulating this behavior is seen in the successful peer-to-peer dialogues (demonstrated in Lesson 1: Clip 1 at 3:30 and Lesson 3: Clip 1 at 1:00, 3:00, 4:50). I also use probing questions with several students to elicit responses about their conceptual understandings related to ratios (demonstrated in Lesson 1: Clip 1 at 6:38).

My approach was to approximate a Socratic dialogue with my students so that I cyclically asked them a question, waited for a response, and then asked a further question until I either had to respond to another student, or the target understanding was achieved (demonstrated in Lesson 1: Clip 1 at 5:40 and Lesson 3: Clip 1 at 5:20). This practice not only helped students develop procedural fluency, but also to justify why each step is valid and necessary, which are essential parts to mathematical reasoning and problem solving.

I deepened student understanding of procedural fluency in several situations by having students explain to me their process (demonstrated in Lesson 1: Clip 1 at 3:30, 7:00 and Lesson 3: Clip 1 0:01, 5:06). This encourages students to review their approach and revise their thinking if an error is detected. (demonstrated in Lesson_3_Clip at 0:19, 6:20).Because of the complexity of the problem, I was able to frequently question students and thereby promote a more thorough conceptual, procedural, mathematical understanding (demonstrated in Lesson 1: Clip 1 at 5:40 and Lesson 3: Clip 1 at 5:20). Additionally student collaborative groups allowed me promote the use of student self-assessment and revision of their own thinking as they worked together to negotiate meaning and make decisions about what the most mathematically reasonable course of action (demonstrated in Lesson 1: Clip 1 at 0:01 and Lesson 3: Clip 1 at 0:10, 2:10, 4:00, 5:00). For example, a group engages is an animated discussion of how to model how people are "zombified" in Lesson 3: Clip 1 at 5:30. Moreover, conceptual understanding was deepened from Lesson 1 to Lesson 3 when I had students use their prior knowledge of "constant multiplier", "starting values" and ratios to a more complex model (demonstrated in Lesson 3:

Clip 1 at 0:01 and 4:00). Finally, my lesson design provided a means of supporting deepening of student learning on a much broader basis than myself working with one student at a time; instead students were able to peer-tutor, which was an intentional aspect of my lesson plan (demonstrated in Lesson 1: Clip 1 at 3:30 and Lesson 3: Clip 1 at 0:45, 4:00, 5:30)

Representations to support understanding

In each of the lessons, I used both planned and student generated representations to support student's understanding and use of mathematical concepts and procedures (as demonstrated in Lesson 1: Clip 1 at 0:25, 1:02, 3:00, 5:15, 6:15 and Lesson 3: Clip 1 at 0:01, 3:00, 3:33, 5:20). To support visual learners, I integrated student-made graphs into the learning task in Lesson 3 (as shown in Lesson 3: Clip 1 at 3:33). This representation challenged students to reason about what might be a useful scale for a population in the hundreds of thousands. Through the lessons, students used multiple graphic representations, including the tables drawn on the desks in both Lesson 1 and Lesson 3, to understand the mathematical patterns and to structure procedures (demonstrated in Lesson 1: Clip 1 at 0:01, 4:50, 5:50 and Lesson 3: Clip 1 at 0:45, 4:00, 5:30). I encouraged one student to use the representation of the table in the book to guide making his own table (Lesson 1: Clip 1 at 3:05). The tables helped students to see patterns, such as the doubling of values in the table at 3:04 (Lesson 3: Clip 1 at 3:00). These representations also make it easier for me to notice conceptual/procedural errors as in Lesson 3: Clip 1 at 3:00. Students both created and identified equations from multiple choice, which requires fluency with mathematical symbols (an example is on the desk in as shown in Lesson 1: Clip 1 at 6:02 and equations appear on the work sheet in Lesson 3: Clip 1 at 3:33).

In our class, we often used the representation "opposites and equity." This is described on a poster as a fulcrum with two equal numbers balanced on it. On several occasions during Lesson 1, I pointed to the poster as a visual representation (as demonstrated in Lesson 1: Clip 1 at 7:25). As a class, we clearly developed this model both as a picture and as helpful saying, with a focus on the academic language of the words "opposite" and "equity. I also used the classroom posters as a resource to stimulate prior knowledge of the procedures related to the order of operations with exponents when students in Lesson 3 used equations to predict future population values. Throughout Lesson 1 and Lesson 2, I encouraged students to collaborate and to share their own personal representations with their team in order to promote group understanding. One real world representation that a student generated was the analogy of ratios as "odds of an event" (Lesson 3: Clip 1 at 6:00). This helped him and his partner to think about how to reduce the ratios. In fact, odds are not something that I have covered in this algebra class, and it was solely because of their interactions that I was able to affirm this alternate way of thinking about ratios. Additionally, I noticed over the period that students responded well to me using hand gestures to represent numbers, so I used this representation to model the values in the problem, and I saw that a student started using this approach to re-explain what I had said to another student. (demonstrated from 5:20 to 6:20 in (Lesson 3: Clip 1).

5. Analysis of teaching

If I were to teach this lesson again, I would scaffold the graphing necessary in the Lesson 3 worksheet to support the individual students with gaps in prior knowledge, like my three students with SLPs, about graphing ordered pairs and choosing a reasonable scale. This also would have benefited the whole class by focusing the activity on new information. It was necessary for students to compare graphs, not create graphs. Second, I would also modify both lessons by utilizing additional higher achieving students to work with under-performing students (as demonstrated in Lesson 3: Clip 1 at 4:00). For example, one of my high-achieving students completed all of the steps in the Lesson 1 investigation and I could have utilized this student to check in with other groups. Specifically, I would modify my assignment for these students by engaging them in peer-tutoring since many of the students who did not want to be videotaped needed more help during the lesson. Third, since many students seemed to enter into disequilibrium while modeling the growth and dying of the zombies in Lesson 3 (Lesson 3: Clip 1 at 0:01 to 3:00) it would have been beneficial to provide an additional visual representation, especially for my student with a 504 for dyslexia. For example, a tree diagram of the zombie population growth, to give students a tool to think about what the growth looks like. Finally, I would have used group roles in each of the lessons, not just Lesson 3. Roles for students in Lesson 1 would have increased the interdependency and collaborative support. This would benefit the whole group because it would provide a clearer structure, and it would benefit individuals who are uncomfortable asking for help.

Changes for improvement

First, I would scaffold the activity for students with prior knowledge gaps in Lesson 3 to support the individual students with gaps in prior knowledge because I identified that once I suggested a scale to struggling students a majority of them could complete the graph. Choosing a scale requires making important conceptual, procedural, and mathematical reasoning connections. I think that this support has the possibility to improve student learning for the whole group as well. Pressley and McCormick (2007) write, “when adults explain, model, and scaffold problem solving for children, they prompt children’s attention to important dimensions of problems” (p. 158). Scaffolding helps students to work on tasks in their zone of proximal development, which is where they are able to complete a task with assistance (Pressley and McCormick, 2007, p. 156).

Second, I would also modify both lessons by utilizing additional higher-achieving students to work with under-performing students because I saw how well this work with several groups, including the one featured at the end of Lesson 3: Clip 1. Research asserts that such peer-tutoring is mutually beneficial (Pressley and McCormick, 2007, p. 276). Specifically, Pressley and McCormick indicate, “learning of the material often is improved more by being a tutor than a tutee” (as cited in Semb, Ellis, & Araujo, 1993). Therefore, facilitating tutoring interactions would increase student learning for the two individuals.

Finally, I would have provided a visual to show the zombie population growth because student struggled modeling only with tables. Cognitive research supports the use of representations that promote mental imaging because “material that is easy to image is recalled more easily” (Pressley and McCormick, 2007, p.114). The addition of a graphical model would address the needs of visual learners. The reason that more diverse types of input is beneficial, according to Medina (2008) is that more information helps people to make more connections with pre-existing knowledge.